Searching for some order in our chaotic world

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Was it a chance encounter that you met that special someone or was there some deeper reason for it? What about that strange dream last night--was that just the random ramblings of the synapses of your brain or was it revealing something deep about your unconscious? Perhaps the dream was trying to tell you something about your future. Perhaps not. Did the fact that a close relative developed a virulent form of cancer have a profound meaning or was it simply a consequence of a random mutation of his DNA? We live our lives thinking about the patterns of events that happen around us. Are they simply random or is there some reason for them?

We can learn something about these types of questions from one of the deepest theorems in mathematical logic. There is much to gain about finding patterns in our life by considering some of the central ideas about finding patterns in a string of characters.

First, some preliminaries. Consider the following three strings of characters:

1. 100100100100100100100100100100100100100100100100100100100100100100100
2. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
3. 38386274868783254735796801834682918987459817087106701409581980418.

How would you describe these strings? One can easily describe them by just writing them down as we just did. However, it is pretty obvious that there are shorter descriptions of the first two strings. The first is simply the pattern "100" over and over. The second pattern is simply a listing of the first few prime numbers. What about the third string? We can describe it by just printing the string. But is there a better, shorter description?

In the early 1960s, an American teenager named Gregory Chaitin, the world famous Russian mathematician Andrey Kolmogorov, and the computer science pioneer Ray Solomonoff independently formulated a way of measuring the complexity of strings of characters. Their ideas have come to be called *Kolmogorov Complexity Theory* or *Algorithmic Information Theory*. They posited that a string is as complex as the length of the shortest computer program that can produce the string. That is, take a string and look for a short computer program that produces that string. The program is a type of description of the string. If the shortest such program is very short, then the string has a simple pattern and is not very complex. We say that string "has little algorithmic content." In contrast, if a long program is required to produce the string, then the string is complicated and "has more algorithmic content." For any string, one must look for the shortest program that produces that string. The length of that program is called *the Kolmogorov complexity of the string*.

Let us look at the above three strings. The first two strings can be described by the relatively short computer programs

1. Print "100" 30 times.
2. Print the first 25 prime numbers.

The Kolmogorov complexity of the first string is less than the Kolmogorov complexity of the second string because the first program is shorter than the second program. What about the third string? There is no obvious pattern for this string. Nevertheless, there exists a silly program that prints this sequence:

3. Print “38386274868783254735796801834682918987459817087106701409581980418”

While this program will do the job, it is not very satisfying. Perhaps there is a shorter program that shows the string has a pattern. When the shortest program to produce a string is simply “Print” the string, we say that the string is very complicated and there is no known pattern. A string that lacks any pattern is called *random*. While we do not see any pattern, there could still be one. This article is concerned with finding patterns to explain the seeming randomness.

We might try to use the amazing powers of modern computers to find a pattern and a shortest program. Wouldn’t it be lovely if there were a computer that would simply calculate the Kolmogorov complexity of any string? This computer would accept a string as input, and output the length of the shortest program that can produce that string. Surely, with all the newfangled computer tools like AI, deep learning, big data, quantum computing, etc., it would be easy to create such a computer. Alas, no such computer can exist! As powerful as modern computers are, this task cannot be accomplished. This is the content of one of the deepest theorems in mathematical logic. Basically, the theorem says that the Kolmogorov complexity of a string cannot be computed. There is no mechanical device to determine the size of the smallest program that produces a given string. It is not that our current level of computer technology is insufficient for the task at hand, or that we are not clever enough to write the algorithm. Rather, it was proven that the very notion of description and computation shows that no such computer can ever possibly perform the task for every string. While a computer might find some pattern in a string, it cannot find the *best* pattern. We might find some short program that outputs a certain pattern, but there could exist an even shorter program. We will never know. A real-world consequence of this fact for the business world is that no one knows if a certain computer program is the most efficient program.

The proof that the Kolmogorov complexity of a sequence is not computable is a bit technical but the ideas behind the proof are very similar to the ideas in two cute little paradoxes. *The interesting number paradox* asks us to determine if a number has an interesting property or not. 1 is the first number, so that is interesting. 2 is the first even number. 3 is the first odd prime number. 4 is interesting because $4=2\times2$ and $4=2+2$. We can continue in this fashion and find interesting properties for many numbers. At some point we might come to some number that does not have an interesting property. We can call that number the first uninteresting number. But that, in itself, is an interesting property. In conclusion, the uninteresting number is, in fact, interesting! This is a seeming contradiction about numbers.

There is a simple resolution to the interesting number paradox. The concept of an interesting number (like that of an uninteresting number) is not well defined. What might be interesting to you will not be interesting to me. Interestingness is not an objective property and hence we cannot make a contradiction about it.

The idea of the proof is also similar to the *Berry paradox*, which is about describing large numbers. Notice that the more words you use, the larger the number you can describe. For example in three words you can describe “a trillion trillion” while in five words you can describe “a trillion trillion trillion trillion” which is much larger. Now consider the number described by the following phrase:

“The smallest number that cannot be described in less than fifteen words.”

This number needs fifteen, or sixteen, or even more words to describe it. It cannot be described by twelve words, or thirteen words, or fourteen words. However there is a major problem: the above phrase described the number in only twelve words. Our description of the number violated the description of the number. This is a contradiction.

The resolution of the Berry paradox is similar to our resolution to the interesting number paradox. There is no objective way to make the connection between numbers and how many words are needed to describe those numbers. Language is not that exact. In both of these paradoxes, we arrive at contradictions by assuming there is an exact way of describing something. Similarly, the way Kolmogorov complexity is not computable transpires from the fact that if it was, we would find a contradiction.

The fact that Kolmogorov complexity is not computable is a result in pure mathematics and one should never confuse that pristine realm with the far more complicated, and messy, real world. However, there are certain common themes about Kolmogorov complexity theory that we might take with us when thinking about the real world. Many times we are presented with something that looks totally chaotic. This randomness is unnerving and so we search for a pattern that eliminates some of the chaos. If we do find a pattern, it is not clear that it is the best pattern that explains what we see. Perhaps there exists a deeper pattern which provides a better explanation. There is no sure way to determine the best pattern. We will simply never know if the pattern that we have is the best.

From the fact that we will never know if the pattern that we have is the best pattern makes the search eternally interesting. By definition, something is interesting if it demands more thought. A fact that is obvious and totally understood does not require further thought. The fact that 6 times 7 is 42 is totally comprehensible and uninteresting. It’s when we are not certain about ideas that we need to confirm them and think about them. The search for better patterns will always be interesting.

There is an added complexity in the real world. Whereas in the world of strings and computer programs there are no mistakes, in the real world we can, and do, make mistakes. We can easily see if a certain program prints out a string or not. While we might not be able to determine the optimal program to print a certain string, we can determine if the program prints the required string. In contrast, the real world is much more complicated. We can think we recognize a pattern when, in fact, we are mistaken. The pattern does not describe the events or phenomena we see. The human machine is imperfect and makes mistakes. This explains why there is a cacophony of opinions at every point of human interaction.

Let us use these insights to comprehend the world of science. A scientist conducts research by examining a seemingly chaotic set of phenomena and looking for a law of nature that would explain those phenomena. The law of nature is really a pattern that explains all the phenomena under scrutiny. Once we find such a law of nature, we are never sure that it is true. The history of science is littered with laws of nature that have been shown to be less than perfect. Either new phenomena will appear that indicate certain anomalies with a law of nature, or another more fundamental law of nature will show that the old law of nature is inadequate. The main point is that scientific knowledge is tentative. As Karl Popper taught us, scientific laws cannot be proven true. Rather they are waiting to be shown false. We cannot say that since a law has been around for a long time it must be true. A law can only be shown to be false or inadequate. This “falsification” is similar to our result about the impossibility of computing the Kolmogorov complexity of a string. This pattern might work, but there might be a better one.

Let us descend from the lofty world of science and see what Kolmogorov complexity tells us about the world around us. When we read a newspaper or a history book, we do not desire a simple list of facts or occurrences. We want analysis. We want to be told what patterns or trends are happening. A good writer will tell us what he thinks about the facts. How they can be put in order. How will these patterns and trends continue into the cloudy future? We are genetically programmed to try to figure out the patterns so that we are better prepared for the future. The inability to be certain about the pattern --- and hence the future --- is what makes us constantly search out for more, and perhaps better, patterns. We will constantly need to put an order on the world around us.

This abhorrence for randomness and the desire for patterns can explain our love of literature, theatre, and the cinema. When we read a novel, or watch a play, the author or director is presenting us with a sequence of events that has a common theme, pattern, or moral. There would be no literary or movie criticism if literature and movies just described random events and did not have some meaning. The author and director are trying to express some coherent ideas about the universe and the human experience. This is exactly what the audience wants. Human beings are biologically programed to find some patterns that explains what they see. It is simple to see such patterns in literature and movies. Most audience members usually go further. Not only do we demand a theme and a moral of the story, we also want a “Hollywood ending.” We want the moral of the story to be positive and uplifting. Literature, plays, and the cinema offer us a delightful escape from the usual unintelligible, meaningless chaos that we find in the real world around us.

What does this tell us about our personal life? While we travel through the seemingly random events in our life, we are searching for patterns, and structure. Life is full of “ups and downs.” There are the joys of falling in love, giggling with your child, and feeling a sense of great accomplishment when a hard job is completed. There is also the pain of a crumbling relationship, or the agony of failing at a task after great effort, or the tragedy of the death of a loved one. We try to make sense of all this. We abhor the feeling of total randomness and idea that we are just following chaotic, habitual laws of physics. We want to know that there is some meaning, purpose, and significance in the world around us. We want a magical story of a life, so we tell ourselves stories. We explain and justify our lives through patterns. Sometimes the stories are simply false. Sometimes we lie to ourselves and those around us. And sometimes our patterns are correct. But even if the story is correct, is the story that we tell the best one? Can there be a deeper story that is more exact? As we age, we gain certain insights about our lives that we did not see before. We are finding better patterns. Maybe we will see things more clearly in the future. Maybe not. We will never know.